Privatization in a Vertical Structure with R&D Investment

Arijit Mukherjee* Xiaoting Wu† Chenhang Zeng‡

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Abstract

We investigate how upstream privatization affects downstream R&D investments and social welfare in a vertically-related market with an upstream monopoly and two downstream firms. One of the downstream firms can undertake R&D investments to increase the utilization efficiency of input good. We consider two different scenarios: a public monopoly and a private monopoly, and show that the downstream firm may have less incentive to invest in the case of a public upstream firm. Furthermore, compared with upstream nationalization, upstream privatization may generate greater social welfare when the innovation is significantly efficient. Our analysis provides implications for privatization policy.

Key words: Vertical structure, R&D investment, Cournot, Upstream privatization, Welfare

JEL Classification: L12, L13, L33

*Nottingham University Business School, Nottingham, UK, Arijit.Mukherjee@nottingham.ac.uk.

†Wenlan School of Business, Zhongnan University of Economics and Law, 182 Nanhu Ave., Wuhan 430073, China

‡Correspondence to: Wenlan School of Business, Zhongnan University of Economics and Law, 182 Nanhu Ave., Wuhan 430073, China, cz_sdu@163.com, Telephone and Fax: +86-027-88387186.

1 Introduction

Public firms are widely observed in many markets in developed, developing, and former communist transitional economies. Typical examples are industries such as airlines, banking, broadcasting, education, electricity, railways, and telecommunications. Over the last decades, whether public firms should be privatized or not has been the focus of debate in both the academic and the political world. The consequences of privatization is still a lively research topic which worthy of study.

The academic literature on mixed oligopolies, such as Harris and Wiens (1980), Beato & Mas-Colell (1984), Cremer et al. (1989), and Barros (1995), believe that a public firm is an effective policy tool in that it can correct the inefficiencies created by the imperfectly competitive market, and therefore favor complete nationalization. However, it has also been shown that privatization could enhance welfare under different market structures. Representative works include Rees (1988), De Fraja & Delbono (1989, 1990), Fershtman (1990), Anderson et al. (1997), Matsumura (1998), and Matsumura & Ogawa (2010), to name but a few. With the consideration of free entry, Matsumura & Kanda (2005) show that while privatization might be optimal in the short run, full nationalization becomes optimal in the long run.

Further, in line with the empirical observation of R&D competition, the theoretical literature also investigate R&D competition between public and private firms. For examples, Delbono & Denicolò (1993), Poyago-Theotoky (1998), Matsumura & Matsushima (2004), Ishibashi & Matsumura (2006), Heywood & Ye (2009), Gil-Moltó et al. (2011), and Gil-Moltó et al. (2020). To discuss the welfare implications of privatization, the papers mentioned above, except Heywood & Ye (2009), examine two different scenarios: mixed duopoly (oligopoly) and private duopoly (oligopoly), and compare the equilibrium results.

The above mentioned works focus on privatization of public firms when they compete with private firms in different frameworks. That is, public and private firms are competitors. Interestingly, the issues of privatization in the context of vertical mixed oligopolies with a public monopoly in the upstream market have been rarely explored, especially with the consideration of downstream R&D. Such market structure with upstream firms in the upstream market and downstream firms in the downstream market is common in many industries. Thus, the study of competitive effects of upstream privatization with R&D activity in the downstream private firm does not only have a purely academic interest but also clear policy relevance.

In this paper, we emphasize this rather neglected aspect in the mixed oligopoly literature, by concentrating on the consequences of upstream privatization with the consideration of downstream R&D. We propose a vertical oligopoly model with an upstream firm supplying input goods to two ex-ante identical downstream firms which engage in Cournot competition. At the beginning of the game, one of the downstream firms has the opportunity to undertake R&D activities which increase its utilization efficiency of input goods. After the realization of R&D, the upstream firm determines the input price, and then downstream firms produce final goods. We solve the game in two different scenarios where the upstream firm can be either a profit-maximizing private firm or a welfare-maximizing public firm, and provide welfare implications of privatization after comparing the equilibrium outcomes in the two scenarios.

We find that privatization in the upstream market helps to increase innovations in the downstream market, and may benefit consumers and social welfare when the efficiency of innovation is significant. The innovation-driven effect of privatization provides an explanation for why we observe more innovations in capitalist countries such as the United States and UK. Furthermore, the result that privatization could benefit consumers is surprising. These findings indicate that privatization could be desirable even if there is a single public firm.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 presents the analysis and main results. Section 4 concludes. Proofs of all lemmas and propositions, as well as derivations for some expressions and claims, are relegated to the Appendix.

2 The Model Setup

Consider a vertically-related market with an upstream firm (firm 0) and two downstream firms (firms 1 and 2). The upstream firm produces an intermediate good at a constant marginal cost denoted by $c(< 1)$. Each downstream producer, firm $i = 1, 2$, purchases the inputs at a wholesale price w, and transforms one unit of input into one unit of a homogeneous product under a constant marginal cost that we normalize to zero. We assume that downstream firms compete in quantities by choosing q_i. In the final goods market, the inverse demand function is given by $P = 1 - Q$, where $Q = q_1 + q_2$ denotes the total outputs.

In our model, we assume that firm 2 can invest K amount in R&D to reduce its input coefficient to λ, where $0 < \lambda < 1$. That is, firm 2 realizes a cost reduction after innovation because it becomes more efficient in production. Then, the two firms profits are given by

$$\begin{cases} \pi_1 = (1 - Q - w)q_1; \\ \pi_2 = (1 - Q - sw)q_2 - R, \end{cases} \tag{1}$$

where $s = \{1, \lambda\}$ and $R = \{0, K\}$. It follows that $R = 0$ when $s = 1$, and $R = K$ when $s = \lambda$. The upstream firm's profit is

$$\pi_0 = (w - c)Q_0, \tag{2}$$

where $Q_0 = q_1 + sq_2$ denotes the total input demand faced by firm 0.

The social welfare, denoted as SW, is the sum of industry profits and consumer surplus (CS):

$$SW = \pi_0 + \pi_1 + \pi_2 + CS, \quad \text{and} \quad CS = Q^2/2. \tag{3}$$

To show the implications of the public provision of inputs, we consider two situations in the following analysis: (i) a profit-maximizing upstream firm, and (ii) a welfare-maximizing upstream firm.

We consider the following three-stage game. In the first stage, firm 2 decides whether or not to invest in R&D. In the second stage, firm 0 determines the input price. In the third stage, firm 1 and firm 2 determine quantities, and then profits are realized. We solve the above game through backward induction.

3 The Analysis and Results

In stage 3, given s and w, firm 1 and firm 2 maximize their individual profits in (1) to determine individual outputs, respectively. The first-order conditions are given by

$$\begin{cases} \partial\pi_1/\partial q_1 = 1 - 2q_1 - q_2 - w = 0; \\ \partial\pi_2/\partial q_2 = 1 - 2q_2 - q_1 - sw = 0. \end{cases} \tag{4}$$

The second-order conditions are satisfied. Solving (4) leads to the equilibrium outputs as

$$q_1^* = \frac{1 - 2w + sw}{3}, \quad \text{and} \quad q_2^* = \frac{1 - 2sw + w}{3}. \tag{5}$$

It follows that $q_1^* \leq q_2^*$. The total input demand is given by

$$Q_0 = q_1^* + sq_2^* = \frac{1 + s - 2w(1-s) - 2s^2w}{3}. \tag{6}$$

Then, the profits of firms 1 and 2 can be obtained as

$$\pi_1^* = \frac{(1 - 2w + sw)^2}{9}, \quad \text{and} \quad \pi_2^* = \frac{(1 - 2sw + w)^2}{9}. \tag{7}$$

In the following, we consider two different cases in which the upstream firm is (i) a profit-maximizing firm in Section 3.1, (ii) a welfare-maximizing firm in Section 3.2.

3.1 The profit-maximizing upstream firm

In stage 2, firm 0 decides the input price w. We first consider the case of a profit-maximizing upstream firm. Then firm 0 determines w to maximize its profit in (2), where Q_0 is obtained in (6). The first-order condition is

$$\frac{\partial\pi_0}{\partial w} = \frac{1 + s + 2(1 - s + s^2)(c - 2w)}{3} = 0,$$

which yields

$$w^* = \frac{2cs^2 - 2cs + 2c + s + 1}{4(s^2 - s + 1)} > 0. \tag{8}$$

The second-order condition is obviously satisfied. Substituting w^* into (5) and (7) yields downstream firms' equilibrium outputs and profits as

$$\begin{cases} q_1^* = \frac{2c\left(s^3-3s^2+3s-2\right)+5s^2-5s+2}{12(s^2-s+1)}; \\ q_2^* = \frac{c\left(2-4s^3+6s^2-6s\right)+2s^2-5s+5}{12(s^2-s+1)}, \end{cases} \tag{9}$$

and

$$\begin{cases} \pi_1^* = \frac{\left(2c\left(s^3-3s^2+3s-2\right)+5s^2-5s+2\right)^2}{144(s^2-s+1)^2}; \\ \pi_2^* = \frac{\left(c\left(2-4s^3+6s^2-6s\right)+2s^2-5s+5\right)^2}{144(s^2-s+1)^2} - R. \end{cases} \tag{10}$$

In stage 1, firm 2 obtains the incentive to invest in R&D if and only if innovation leads to a higher profit than non-innovation does, i.e., $\pi_2^*|_{s=\lambda} > \pi_2^*|_{s=1}$, which leads to

$$K < \frac{\left(c(-4\lambda^3 + 6\lambda^2 - 6\lambda + 2) + 2\lambda^2 - 5\lambda + 5\right)^2}{144(\lambda^2 - \lambda + 1)^2} - \frac{(2(1-c))^2}{144} \equiv \overline{K}, \tag{11}$$

where $\overline{K} > 0$, which indicates that innovation occurs in this case as long as the cost of innovation is sufficiently small. Replacing s with λ and R with K in both (9) and (10) leads to the equilibrium outputs and profits under innovation. We assume

$$0 < c < \frac{2 + 5\lambda^2 - 5\lambda}{4 - 2\lambda^3 + 6\lambda^2 - 6\lambda} \equiv \bar{c}(\lambda), \tag{12}$$

such that both downstream firms are active in equilibrium.

Note that $\bar{c}(\lambda)$ decreases with λ if $\lambda \in (0, 1/2)$ and increases with λ if $\lambda \in (1/2, 1)$ (see Figure 1). Further, we have $\bar{c}(1) = 1$ and $\bar{c}(0) = 1/2$.

3.2 The welfare-maximizing upstream firm

Consider firm 0 is a welfare maximizing firm in this case. Then in stage 2, firm 0 determines w to maximize the social welfare in (3). The first-order condition is

$$\frac{\partial SW}{\partial w} = \frac{6c\left(s^2 - s + 1\right) - (s+1)(sw + w + 1)}{9} = 0,$$

which yields

$$w = \frac{6cs^2 - 6cs + 6c - s - 1}{(s+1)^2}.$$

The second-order condition is obviously satisfied. Note that w in the above equation could be negative when c is sufficient small, i.e.,

$$0 < c \leq \frac{s+1}{6(s^2 - s + 1)} \equiv \hat{c}(s),$$

where $\hat{c}(s)$ increases with s if $s \in (0, \sqrt{3} - 1)$ and decreases with s if $s > \sqrt{3} - 1$. Further, we have $\hat{c}(1/2) = \hat{c}(1) = 1/3$ (see Figure 1).

A negative input price implies that a per-unit subsidy is provided to downstream firms, which causes downstream firms to keep purchasing inputs without using them. We then restrict the input price to be non-negative in our analysis. The equilibrium input price is then given by

$$w^{**} = \begin{cases} 0, & \text{when } 0 < c \leq \hat{c}(s); \\ (6cs^2 - 6cs + 6c - s - 1)/(s+1)^2, & \text{when } \hat{c}(s) < c < 1. \end{cases} \tag{13}$$

When $0 < c \leq \hat{c}(s)$, applying $w^{**} = 0$ into (5) and (7) leads to

$$q_1^{**} = q_2^{**} = \frac{1}{3}; \quad \text{and} \quad \pi_1^{**} = \frac{1}{9}, \quad \pi_2^{**} = \frac{1}{9} - R. \tag{14}$$

Similarly, when $\hat{c}(s) < c < 1$, applying $w^{**} = \frac{6cs^2-6cs+6c-s-1}{(s+1)^2}$ into (5) and (7) leads to

$$\begin{cases} q_1^{**} = \frac{2c\left(s^3-3s^2+3s-2\right)+s+1}{(s+1)^2}; \\ q_2^{**} = \frac{c\left(-4s^3+6s^2-6s+2\right)+s(s+1)}{(s+1)^2}, \end{cases} \tag{15}$$

and

$$\begin{cases} \pi_1^{**} = \frac{\left(2c\left(s^3-3s^2+3s-2\right)+s+1\right)^2}{(s+1)^4}; \\ \pi_2^{**} = \frac{\left(c\left(-4s^3+6s^2-6s+2\right)+s(s+1)\right)^2}{(s+1)^4} - R. \end{cases} \tag{16}$$

We assume

$$0 < c < \widetilde{c}\,(s) \equiv \frac{s+1}{4-2s^3+6s^2-6s}$$

such that both downstream firms are active in equilibrium, where $\widetilde{c}\,(s) > \hat{c}(s)$. Otherwise, firm 1 will be driven out of market. Note that $\widetilde{c}(s)$ increases with s for $s \in (0,1)$; furthermore, $\widetilde{c}(0) = 1/4$ and $\widetilde{c}(1) = 1$ (see Figure 1).

In stage 1, firm 2 decides whether or not to make R&D investments. As in the previous case, firm 2 invests in R&D if and only if $\pi_2^{**}|_{s=\lambda} > \pi_2^{**}|_{s=1}$, in which π_2^{**} critically depends on the value of input price as discussed above. We use Scenario 1 and Scenario 2 to represent the situations in which firm 2 decides to invest ($s = \lambda$) and not to invest ($s = 1$), respectively. Since the input price in each scenario can be either zero or positive, we conduct the comparison in the following four different cases.

Case 1: The equilibrium input prices are positive in both scenarios.

This case occurs when $\max\{\hat{c}(1), \hat{c}(\lambda)\} < c < \min\{\widetilde{c}(1), \widetilde{c}(\lambda)\}$. Then, we obtain firm 2's profit in each scenario by substituting $s = \lambda$ and $s = 1$ into (16), respectively. Firm 2 will choose to invest if the investment is profitable, which requires

$$K < \frac{\left(2c\left(-2\lambda^3+3\lambda^2-3\lambda+1\right)+\lambda(\lambda+1)\right)^2}{(\lambda+1)^4} - \frac{(1-c)^2}{4} \equiv \widetilde{K}_1,$$

where $\widetilde{K}_1 > 0$. The above inequality indicates that firm 2 will choose to invest as long as the innovation cost is not too high. Under innovation, the equilibrium outputs and profits of firms 1 and 2 can be obtained by replacing s with λ, and R with K in (15) and (16).

Case 2: The equilibrium input price in Scenario 1 is positive while that in Scenario 2 is zero.

This case occurs when $\max\{0, \hat{c}(\lambda)\} < c < \min\{\hat{c}(1), \widetilde{c}(\lambda)\}$.In Scenario 1 in which firm 2 makes the investments, we obtain the profit of firm 2 by substituting $s = \lambda$ into (16). In Scenario 2 in which firm 2 does not make the investments, we obtain the profit of firm 2 by substituting $R = 0$ into (14). Hence, firm 2 will choose to invest if and only if

$$K < \frac{\left(2c\left(-2\lambda^3 + 3\lambda^2 - 3\lambda + 1\right) + \lambda(\lambda + 1)\right)^2}{(\lambda + 1)^4} - \frac{1}{9} \equiv \widetilde{K}_2,$$

where $\widetilde{K}_2 > 0$. Hence, firm 2 will choose to invest as long as the innovation cost is not too high. Similarly, under innovation, we obtain the equilibrium outputs and profits of firms 1 and 2 by replacing s with λ, and R with K in (15) and (16).

Case 3: The equilibrium input price in Scenario 1 is zero while that in Scenario 2 is positive.

This case occurs when $\max\{0, \hat{c}(1)\} < c < \min\{\hat{c}(\lambda), \widetilde{c}(1)\}$.In Scenario 1, we obtain the profit of firm 2 by substituting $R = K$ into (14). In Scenario 2, we obtain the profit of firm 2 by substituting $s = 1$ and $R = 0$ into (16). Hence, firm 2 will choose to invest if and only if

$$K < \frac{1}{9} - \frac{(1 - c)^2}{4} \equiv \widetilde{K}_3,$$

where $\widetilde{K}_3 > 0$. Similarly, firm 2 will choose to invest when the innovation cost K is not too high. Under innovation, we obtain the equilibrium outputs and profits of firms 1 and 2 by replacing R with K in (14).

Case 4: The equilibrium input prices are zero in both scenarios.

This case occurs when $0 < c \leq \min\{\hat{c}(1), \hat{c}(\lambda)\}$, which corresponds to Region III, and Region IV in Figure 1 that presented in Section 3.3. It follows straightforwardly from (14) that firm 2 will not choose to invest as long as there is a positive innovation cost. Then the equilibrium outputs and profits of firm 1 and 2 are

$$q_1^{**} = q_2^{**} = \frac{1}{3}, \quad \text{and} \quad \pi_1^{**} = \pi_2^{**} = \frac{1}{9}.$$

3.3 The comparisons: upstream privatization vs. upstream nationalization

In this section, we conduct several comparisons between the case of a private upstream monopoly and that of a public upstream monopoly to see the implications of privatization. For comparison purposes, we make the following two assumptions: (i) $K < \min\{\overline{K}, \widetilde{K}_1, \widetilde{K}_2, \widetilde{K}_3\}$; and (ii) $c < \max\{\bar{c}(\lambda), \widetilde{c}(\lambda)\}$. The former assumption on K ensures that innovation will occur if firm 2's operating profit in stage 3 (i.e., the final profit excludes the innovation cost) is higher under innovation. The latter assumption on c indicates that all the regions in which both firms are active in the previous cases are included.

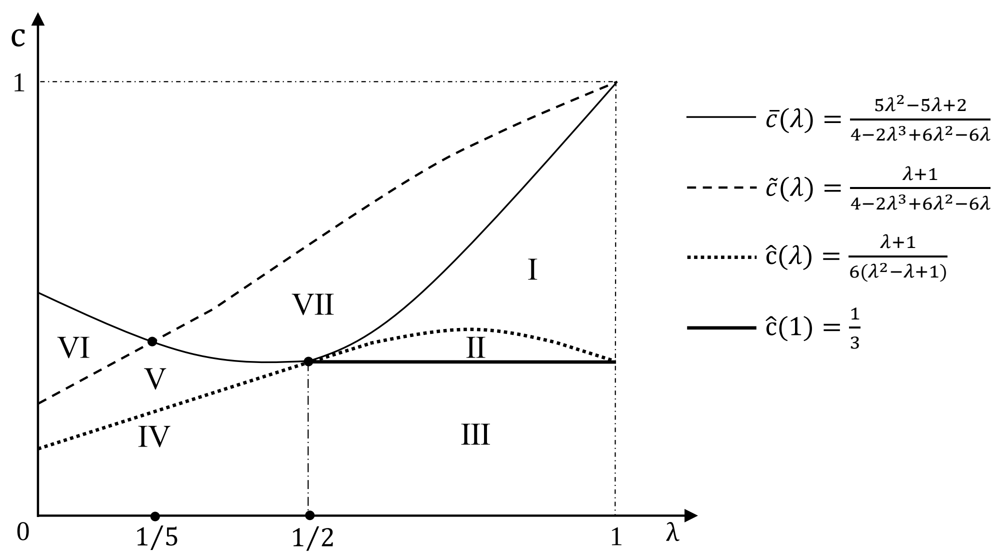

Figure 1: Innovation, Output, and Welfare

Based on the results in the previous two sections, we can plot the following figure to show our main findings innovation, total outputs, and social welfare. In Figure 1,

- The black line $\bar{c}\,(\lambda)$ guarantees that both firms are active in the case of upstream privatization. Since $K < \overline{K}$, firm 2 chooses to invest in all the regions below the black line $\bar{c}\,(\lambda)$.

Furthermore, in Region VII (see the equilibrium results in Appendix), firm 2 invests on innovation and drives firm 1 out of the market.

- The dashed line $\widetilde{c}\,(\lambda)$ guarantees that both firms are active in the case of upstream nationalization. The bold horizontal line denotes $c = 1/3$. And the dotted line $\hat{c}(\lambda)$ is critical for the sign of the input price, above (below) which we have positive (zero) input prices. Based on the analysis of the four cases in Section 3.2, we obtain that firm 2 will choose to invest in regions I, II, V, VII. Furthermore, in Region VI (see the equilibrium results in Appendix), firm 2 invests on innovation and drives firm 1 out of the market.

- Note that (i) the two lines, $\overline{c}\,(\lambda)$ and $\widetilde{c}\,(\lambda)$, intersect at $\lambda = 1/5$, and (ii) the three lines, $\overline{c}\,(\lambda)$, $\hat{c}(\lambda)$, and $\hat{c}(1)$, intersect at $\lambda = 1/2$.

We then summarize our results in the following proposition.

Proposition 1. *In equilibrium, (i) if the upstream monopoly is a private firm, firm 2 will choose to invest in regions I, II, III, IV, V, VI, VII; and (ii) if the upstream monopoly is a public firm, firm 2 will choose to invest in regions I, II, V, VI, VII.*

Proposition 1 implies that innovation occurs in a wider range of parameters when the upstream monopoly is a private firm. That is, privatization in the upstream market helps to increase the likelihood of innovation in the downstream market.

With a public firm in the upstream market, firm 2 obtains no incentive to innovate when the upstream firm is very efficient, i.e., c is small, regardless of the efficiency of innovation. This is because that this efficient upstream public firm always provides the input goods for free aiming to stimulate final goods production. Hence, firm 2 has no incentive to innovate to increase the utilization efficiency of input goods.

The result in Proposition 1 also explains why we observe higher innovation in capitalist countries provided the empirical evidences on innovation in the United States, UK, etc.

When the upstream firm aims to maximize welfare, it would be induced to encourage the production of final goods through input pricing. Thus, we naturally conjecture that a welfare-maximizing upstream firm charges a lower input price compared to a profit-maximizing upstream firm. That is, privatization raises the input price in equilibrium. This is true in most of the regions in Figure 1. However, we identify in Lemma 1 that privatization may lead to a lower input price when c is large and λ is small (i.e., in Region VI and part of Region V).

Lemma 1. *In equilibrium, $w^* < w^{**}$ in Region VI, and Region V when $\lambda < 1/5$ and $c_q < c < \widetilde{c}(\lambda)$; otherwise, $w^* > w^{**}$, where*

$$c_q = \frac{5\lambda^3 + 3\lambda^2 + 3\lambda + 5}{22\lambda^4 - 50\lambda^3 + 72\lambda^2 - 50\lambda + 22}.$$

The reason for $w^* < w^{**}$ in Region VI is straightforward since firm 2 monopolizes the downstream market under upstream nationalization which then leads to a higher input price. In Region V with a sufficiently low λ (i.e., $\lambda < 1/5$), if the upstream firm aims to maximize its profit, it has a strong incentive to charge a low input price to reduce the difference in downstream firms' marginal costs, $\Delta = (1 - \lambda)w$, so as to intensify the competition and increase input demand. As long as the welfare-maximizing upstream firm is not very aggressive in input pricing (i.e., c is relatively large), the profit-maximizing upstream firm will charge a lower input price given its incentive to reduce the input price.

The results in Lemma 1 automatically lead to the results in Proposition 2. If $w^* < w^{**}$, then Δ is smaller with a profit-maximizing upstream firm, which implies that the two downstream firms compete more aggressively and therefore leads to higher total outputs in equilibrium.

Proposition 2. *In equilibrium, (i) $Q^* < Q^{**}$ in Regions I, II, III, IV, VII; (ii) $Q^* > Q^{**}$ in Region VI; and (iii) $Q^* > Q^{**}$ in Region V when $\lambda < 1/5$ and $c_q < c < \widetilde{c}(\lambda)$, otherwise, $Q^* < Q^{**}$.*

Proposition 2 demonstrates the possibility that privatization could benefit consumers when the innovation efficiency is high. The findings in Proposition 1 and 2 indicate that privatization could be desirable even if there is a single public firm.

We next study how privatization changes the industry profits, $\Pi = \pi_0 + \pi_1 + \pi_2$.

Proposition 3. *In equilibrium, (i)* $\Pi^* > \Pi^{**}$ *in Regions I, II, III, IV; (ii)* $\Pi^{**} > \Pi^*$ *in Region VI; (iii)* $\Pi^{**} > \Pi^*$ *in Region V when* $\lambda < 1/5$ *and* $c_q < c < \widetilde{c}(\lambda)$*, otherwise,* $\Pi^{**} < \Pi^*$*; and* $\Pi^* > \Pi^{**}$ *in Region VII when* $0.27 < \lambda < 1$ *and* $\bar{c}(\lambda) \leq c < c_\pi$*, otherwise,* $\Pi^* < \Pi^{**}$*, where*

$$c_\pi = \frac{11\lambda^2 - 13\lambda + 8}{3\lambda^2(\lambda + 1)} - \frac{4}{3}\sqrt{\frac{7\lambda^4 - 19\lambda^3 + 21\lambda^2 - 13\lambda + 4}{\lambda^4(\lambda + 1)^2}}.$$

Proposition 3

To end the analysis, we calculate the social welfare in equilibrium following (3). A straightforward comparison of social welfare in the two cases, upstream privatization and upstream nationalization, gives the following result.

Proposition 4. *In equilibrium, (i)* $SW^* > SW^{**}$ *occurs in Region IV when* $\lambda < 1/2$ *and* $c_{sw} < c \leq \hat{c}(\lambda)$*, and (ii) otherwise,* $SW^* < SW^{**}$*, where*

$$c_{sw} = \frac{17\lambda^3 - 45\lambda^2 + 51\lambda - 31}{2\left(\lambda^2 - \lambda + 1\right)\left(23\lambda^2 - 26\lambda + 23\right)} + 2\sqrt{\frac{31\lambda^4 - 62\lambda^3 + 144\lambda^2 - 122\lambda + 73}{\left(\lambda^2 - \lambda + 1\right)\left(23\lambda^2 - 26\lambda + 23\right)^2}}.$$

Proposition 4 highlights the possibility that upstream privatization may generate greater social welfare than upstream nationalization when K is sufficiently small. The reasons are as follows. In Region IV, the innovation is efficient (i.e., $\lambda < 1/2$), and the profit-maximizing upstream firm charges a higher input price compared to a welfare-maximizing upstream firm. As a result, $\Delta = (1 - \lambda)w$ is higher under upstream privatization, which may help the efficient firm (firm 2) to achieve a dominant position in the final goods market. This increases industry profits, and is beneficial to social welfare. And for the case of upstream nationalization, the upstream firm suffers a loss in profit due to the zero input pricing, especially when the marginal cost c is relatively large. Hence, when c is relatively large, the improvement in industry profits under upstream privatization may lead to greater social welfare than upstream nationalization, though upstream privatization lowers the total outputs and consumer surplus.

4 Concluding Remarks

In this paper, we investigate how upstream privatization affects downstream $R\&D$ investments and social welfare in a vertically-related market, where there is an upstream monopoly and two downstream firms. One of the downstream firms can undertake $R\&D$ investments to increase the utilization efficiency of input good and these two firms compete in quantity. We consider two different scenarios: a public monopoly and a private monopoly, and show that the downstream firm may have less incentive to invest in the case of a public upstream firm. Furthermore, compared with upstream nationalization, upstream privatization may generate greater social welfare when the innovation is significantly efficient. Our analysis provides implications for privatization policy.

Appendix

Derivations of equilibrium results in Region VII

In the case of a profit-maximizing upstream firm, firm 2 invests on innovation and drives firm 1 out of the market in Region VII. Then firm 2 monopolizes the downstream market, and determines its output to maximize $\pi_2 = (1 - q_2 - \lambda w)q_2 - K$. The first-order condition is $\partial \pi_2/\partial q_2 = 1 - 2q_2 - \lambda w = 0$, which yields $q_2^* = (1 - \lambda w)/2$. The second-order condition is obviously satisfied.

Then firm 0 determines the input price to maximize its own profit $\pi_0 = (w - c)Q_0$, such that $q_1^* = (1 - 2w^* + \lambda w^*)/3 \leq 0$, i.e., firm 1 will be driven out of the market. It follows $Q_0 = \lambda q_2^*$. Then the first-order condition is $\partial \pi_0/\partial w = (\lambda - \lambda^2(w - c))/2 = 0$, which yields

$$w^* = \frac{c\lambda + 1}{2\lambda}.$$

The second-order condition and the constraint $q_1^* \leq 0$ are satisfied. Then we obtain

$$q_2^* = \frac{1 - c\lambda}{4} \quad , \quad \pi_2^* = \frac{(c\lambda - 1)^2}{16} - K \quad \text{and} \quad \pi_0^* = \frac{(c\lambda - 1)^2}{8}.$$

Derivations of equilibrium results in Region VI

In the case of a welfare-maximizing upstream firm, firm 2 invests on innovation and drives firm 1 out of the market in Region VI. Then firm 2 monopolizes the downstream market, and determines its output to maximize its profit. Following the previous analysis, we obtain $q_2^{**} = (1 - \lambda w)/2$.

Then firm 0 determines the input price by maximizing $SW = \pi_0 + \pi_2 + CS$, such that $q_1^{**} = (1 - 2w + \lambda w)/3 \leq 0$, where $\pi_0 = (w - c)\lambda q_2^{**}$ and $CS = (q_2^{**})^2/2$. The first-order condition is $\partial SW/\partial w = \lambda(2c\lambda - \lambda w - 1)/4 = 0$, which yields $w = (2c\lambda - 1)/\lambda < 0$. The second-order condition holds. Recall that $q_1^{**} \leq 0$, which requires $w \geq 1/(2 - \lambda)$. Then we obtain

$$w^{**} = \frac{1}{2-\lambda},$$

which yields

$$q_2^{**} = \frac{\lambda-1}{\lambda-2} \quad , \quad \pi_2^{**} = \frac{(\lambda-1)^2}{(\lambda-2)^2} - K \quad \text{and} \quad \pi_0^{**} = \frac{(1-\lambda)\lambda(c(\lambda-2)+1)}{(\lambda-2)^2}.$$

Proof of Lemma 1

In the case of a profit-maximizing upstream firm, we have

$$w^* = \begin{cases} \frac{2c\lambda^2-2c\lambda+2c+\lambda+1}{4(\lambda^2-\lambda+1)}, & \text{in Regions I - VI} \\ \frac{c\lambda+1}{2\lambda}. & \text{in Region VII} \end{cases}$$

In the case of a welfare-maximizing upstream firm, we have

$$w^{**} = \begin{cases} \frac{6c\lambda^2-6c\lambda+6c-\lambda-1}{(\lambda+1)^2}, & \text{in Regions I, V, VII} \\ \frac{1}{2-\lambda}, & \text{in Region VI} \\ 0. & \text{in Regions II, III, IV} \end{cases}$$

Straightforward comparisons lead to the following results: (i) in Regions I, II, III, IV and VII, it follows immediately that $w^* > w^{**}$; (ii) in Region V, we obtain $w^* < w^{**}$ if and only if $0 < \lambda < 1/5$ and $c_q < c < \widetilde{c}(\lambda)$, where c_q is given in Lemma 1; and (iii) in Region VI, we obtain $w^* < w^{**}$.

Proof of Proposition 2

From (5), we have $Q = q_1 + q_2 = (2 - sw - w)/3$, which decreases with w. Based on Lemma 1, we immediately obtain the results in Proposition 2.

Proof of Proposition 3

From (6) and (7), we obtain industry profits as

$$\Pi = \frac{(1-2w+sw)^2}{9} + \frac{(1-2sw+w)^2}{9} + \frac{(w-c)(1+s-2w(1-s)-2s^2w)}{3} - R$$

Given the innovation decisions in Proposition 1 which determine s and R, and the input price in the proof of Lemma 1, we obtain

$$\Pi^* = \begin{cases} \frac{4(\lambda(11\lambda-14)+11)(c(\lambda-1)\lambda+c)^2+F(\lambda)}{144((\lambda-1)\lambda+1)^2} - K, & \text{in Regions I - VI} \\ \frac{3(c\lambda-1)^2}{16} - K, & \text{in Region VII} \end{cases}$$

where $F(\lambda) = (\lambda(5\lambda-2)+5)(\lambda(7\lambda-10)+7) - 4c(\lambda+1)((\lambda-1)\lambda+1)(\lambda(5\lambda-2)+5)$, and

$$\Pi^{**} = \begin{cases} \frac{c(\lambda-1)^2}{\lambda+1} - K, & \text{in Regions I, V, VII} \\ \frac{2-3c(\lambda+1)}{9} - K, & \text{in Region II} \\ \frac{2-6c}{9}, & \text{in Regions III, IV} \\ \frac{(1-\lambda)(c(\lambda-2)\lambda+1)}{(\lambda-2)^2} - K. & \text{in Region VI} \end{cases}$$

Then straightforward comparisons lead to the following results.

In Regions I, II, we have $\Pi^* > \Pi^{**}$.

In Regions III, IV, we have $\Pi^* > \Pi^{**}$ as long as the innovation cost K is not sufficiently high, i.e., $K < \Pi^* - \Pi^{**}$.

In Region V, we have $\Pi^{**} > \Pi^*$ if and only if $0 < \lambda < 1/5$ and $c_q < c < \widetilde{c}(\lambda)$, where c_q is given in Lemma 1.

In Region VI, we have $\Pi^{**} > \Pi^*$.

In Region VII, we have $\Pi^* > \Pi^{**}$ if and only if $0.27 < \lambda < 1$ and $\bar{c}(\lambda) \leq c < c_\pi$, where 0.27 is the unique real root of $307\lambda^5 - 661\lambda^4 + 1072\lambda^3 - 872\lambda^2 + 464\lambda - 80 = 0$.

Proof of Proposition 4

Following (3), simple calculations lead to the social welfare in the two scenarios as

$$SW^* = \begin{cases} \frac{4c^2(\lambda^2-\lambda+1)^2(23\lambda^2-26\lambda+23)+H(\lambda)}{288(\lambda^2-\lambda+1)^2} - K, & \text{in Regions I - VI} \\ \frac{7(c\lambda-1)^2}{32} - K, & \text{in Region VII} \end{cases}$$

where $H(\lambda) = 119\lambda^4 - 268\lambda^3 + 378\lambda^2 - 268\lambda + 119 - 4c(17\lambda^5 - 14\lambda^4 + 17\lambda^3 + 17\lambda^2 - 14\lambda + 17)$, and

$$SW^{**} = \begin{cases} \frac{4c^2\left(\lambda^2-\lambda+1\right)^2 - 2c\left(\lambda^3+\lambda^2+\lambda+1\right)+(\lambda+1)^2}{2(\lambda+1)^2} - K, & \text{in Regions I, V, VII} \\ \frac{4-3c(1+\lambda)}{9} - K, & \text{in Region II} \\ \frac{4-6c}{9}, & \text{in Regions III, IV} \\ \frac{(1-\lambda)(\lambda(2c(\lambda-2)-1)+3)}{2(\lambda-2)^2} - K. & \text{in Region VI} \end{cases}$$

Then straightforward comparisons lead to the following results. (i) in Regions I, II, III, V, VI and VII, we have $SW^* \leq SW^{**}$; (ii) in Region IV, $SW^* > SW^{**}$ occurs if $0 < \lambda < 1/2$, $c_{sw} < c \leq \hat{c}(\lambda)$, and $K < SW^* - SW^{**}$.

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